# Word Representation II: Statistical Methods

Lecture 5



### Announcements

- Late submission by a few minutes:
  - No problem. Just don't do it for HW2.
- HW2 will be posted soon (today or tomorrow).

- One more lecture this Thursday, two PSs next week.
  - Today: Language Model before Transformer.
  - Thursday: Transformers.
  - Next 4 PSs: Hovhannes walks through the entire Transformer architecture.

### Outline

- Global Word Embeddings
  - Word2vec
  - GloVe
- Contextual Word Embeddings
  - Language Modelling
  - RNN

### Softmax

- Say now we have a lot of logits:  $\mathbf{x} = [x_1, \dots x_n], x_i \in (-\infty, +\infty)$
- We want a probability distribution that
  - $\mathbf{p} = f(\mathbf{x}) = [p_1, \dots p_n] = f([x_1, \dots, x_n]) = [f(x_1), \dots, f(x_n)]$
  - Probability distribution -> everything sums to 1.  $\sum \mathbf{p} = 1$
  - Differentiable everywhere.
- Solution:

Softmax
$$(x_i) = \frac{\exp(x_i)}{\sum_j \exp(x_j)}$$

# Representing Data

- Earlier success in computer vision.
  - Navlab 5 (Jochem et al., 1995)





• Much more intuitive to convert images into vector representations.

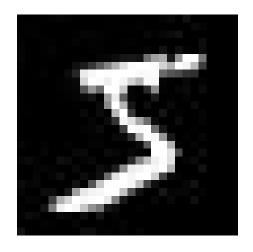
# Representing Data

- Numeric Data:
  - E.g. credit score:

	Number Of Open	Number Of	Number Real		
Monthly	Credit Lines And	Times 90 Days	Estate Loans Or	Number Of Time 60-89	Number Of
Income	Loans	Late	Lines	Days Past Due Not Worse	Dependents
9120	13	0	6	0	2
2600	4	0	0	0	1
3042	2	1	0	0	0
3300	5	0	0	0	0
63588	7	0	1	0	0
3500	3	0	1	0	1
NA	8	0	3	0	0
3500	8	0	0	0	0

# Representing Data

- Numeric Data:
  - E.g. credit score:
- Images:
  - Gray scale or RGB



MNIST dataset Handwritten numbers

- Videos:
  - Images on a timeline





## Representing Textual Data

- The vast majority of rule--based and statistical NLP work regarded words as atomic symbols. Det → the | a | an
  - Recall Lecture 3:

```
Det \rightarrow the | a | an

Adj \rightarrow old | red | happy | ...

N \rightarrow dog | park | ice-cream | contumely | run | ...

V \rightarrow saw | ate | run | disdained | ...

P \rightarrow in | to | on | under | with | ... }
```

Vector space: this is a vector with one 1 and a lot of zeroes:

- The "one-hot" representation
- i-th word in the dictionary:

$$v_i = 1, \forall j \neq i, v_j = 0$$

## Representing Textual Data: Problems

- There are a lot of words!
  - Oxford English Dictionary: 500,000+ entries
  - Longman Dictionary of Contemporary English: 230,000 words
  - Brysbaert et al. (2016): 42,000 lemmas
  - As a result, a lot of BIG vectors!
  - For reference, (L)LM dimensions:
    - BERT, GPT-2: 768
    - Llama-3-8B: 4096
    - Llama-3.1-405B: 16384
- No useful similarity information:
  - Motel: [0 0 0 0 0 0 0 0 0 1 0 0 0 0]
  - Hotel: [0 0 0 0 0 0 0 1 0 0 0 0 0 0]

- Cosine similarity measures the cosine of the angle between two vectors.
- Inner product normalized by the vector lengths.

CosSim(
$$\mathbf{d}_{j}, \mathbf{q}$$
) =  $\frac{\vec{d}_{j} \cdot \vec{q}}{\left|\vec{d}_{j}\right| \cdot \left|\vec{q}\right|} = \frac{\sum_{i=1}^{t} (w_{ij} \cdot w_{iq})}{\sqrt{\sum_{i=1}^{t} w_{ij}^{2} \cdot \sum_{i=1}^{t} w_{iq}^{2}}}$ 

$$\begin{array}{ll} D_1 = 2T_1 + 3T_2 + 5T_3 & \text{CosSim}(D_1 \text{ , } Q) = 10 \text{ / } \sqrt{(4+9+25)(0+0+4)} = 0.81 \\ D_2 = 3T_1 + 7T_2 + 1T_3 & \text{CosSim}(D_2 \text{ , } Q) = 2 \text{ / } \sqrt{(9+49+1)(0+0+4)} = 0.13 \\ Q = 0T_1 + 0T_2 + 2T_3 & \end{array}$$

 $D_1$  is 6 times better than  $D_2$  using cosine similarity but only 5 times better using inner product.

• cos\_sim(motel, hotel)=0, cos\_sim(motel, linguist)=0, cos\_sim(hotel, linguist)=0

### Distributional similarity based representations

- You can get a lot of value by representing a word by means of its neighbors:
- "Noscitur a sociis"
  - The meaning of an unclear or ambiguous word should be determined by considering the words with which it is associated in the context.
  - 19th-century rule of interpretation in English civil courts.
- One of the most successful ideas of modern NLP

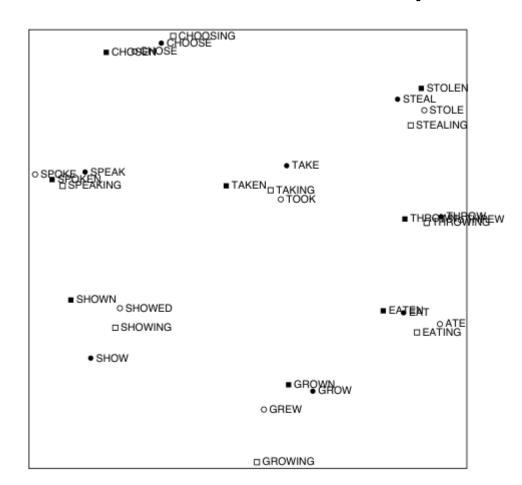
government debt problems turning into banking crises as has happened in saying that Europe needs unified banking regulation to replace the hodgepodge

These words will represent banking 7

# With distributed, distributional representations, syntactic and semantic information can be captured

0.286 0.792 -0.177 -0.107 0.109 -0.542 0.349 0.271

Synonymy? Hyponymy? Morphology?



[Rohde et al. 2005. An Improved Model of Semantic Similarity Based on Lexical Co-Occurrence]

### Two Kinds of Vectors

#### • Count:

- tf-idf, PMI, LSA
- Sparse!
- Information Retrieval workhorse!
- Words are represented by (a simple function of) the counts of nearby words

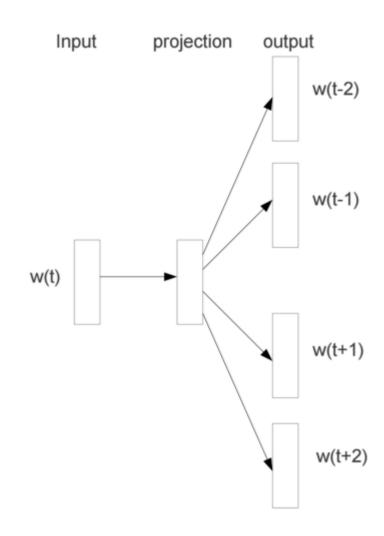
#### • Predict:

- word2vec, GloVe, BERT, GPT-2, GPT-3, GPT-4...
- Dense!
- Representation is created by training a classifier to predict whether a word is likely to appear nearby
- Contextual embeddings.

### Word2vec

### Word2vec CBOW/SkipGram: Predict!

- Train word vectors to try to either
  - Predict a word given its bag-of-words context (CBOW); or
  - Predict a context word (positionindependent) from the center word
- Update word vectors until they can do this prediction well



# Skip-Gram Training Data

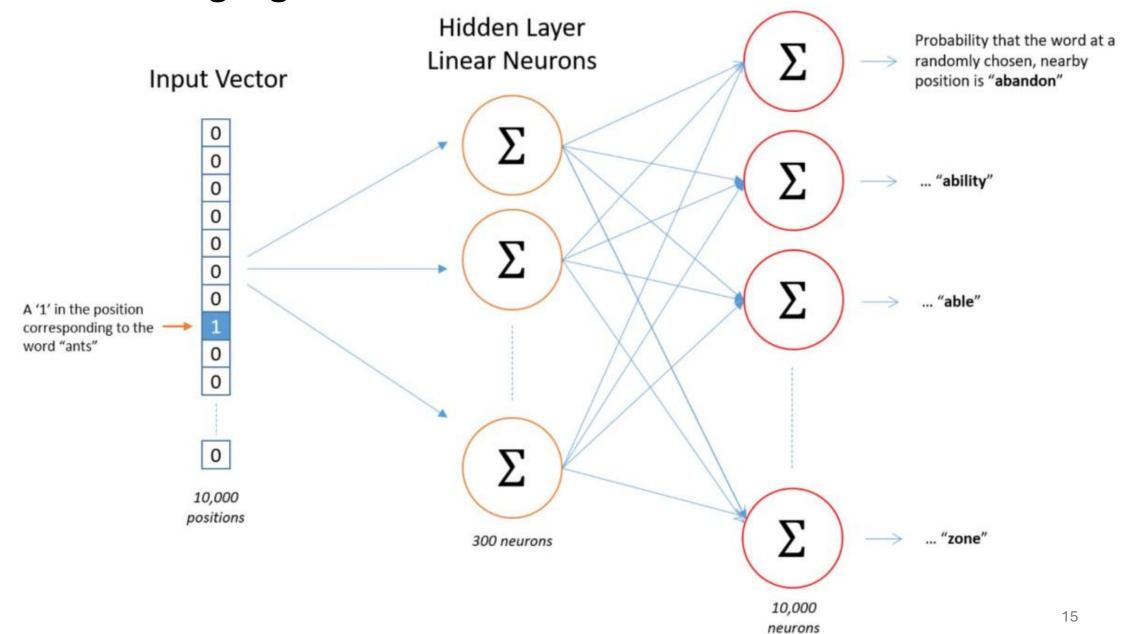
• Assume a +/- 2 word window, given training sentence:

```
...lemon, a [tablespoon of apricot jam, a] pinch
c1 c2 [target] c3 c4
```

- Goal: train a classifier that is given a candidate (word, context) pair
- And assigns each pair a probability:
  - P(+|w, c)
  - P(-|w, c) = 1 P(+|w, c)

### Word2vec training regimen

### Output Layer Softmax Classifier



```
class Word2Vec(nn.Module):
   def init (self, vocab size, embedding size):
        super().__init__()
        self.embed = nn.Embedding(vocab_size, embedding_size)
        self.expand = nn.Linear(embedding_size, vocab_size, bias=False)
   def forward(self, input):
       # Encode input to lower-dimensional representation
       hidden = self.embed(input)
       # Expand hidden layer to predictions
        logits = self.expand(hidden)
        return logits
```

### Approach: predict if candidate word c is a "neighbor"

- 1. Treat the target word *t* and a neighboring context word *c* as **positive examples**.
- 2. Randomly sample other words in the lexicon to get negative examples.
- 3. Use logistic regression to train a classifier to distinguish those two cases.
- 4. Use the learned model activations as the embeddings.

# Word Analogies: word2vec captures dimensions of similarity as linear relations

Test for linear relationships, examined by Mikolov et al. (2013)

man:woman :: king:? [0.30 0.70] queen king 0.75 \* king [0.200.20]man 0.5 [0.600.30]woman woman 0.25 **∞** man [0.700.80]queen 0 0.25 0.5 0.75

# Word Analogies [Mikolov et al., 2012, 2013]

Type of relationship	Word Pair 1		Word Pair 2	
Common capital city	Athens	Greece	Oslo	Norway
All capital cities	Astana	Kazakhstan	Harare	Zimbabwe
Currency	Angola	kwanza	Iran	rial
City-in-state	Chicago	Illinois	Stockton	California
Man-Woman	brother	sister	grandson	granddaughter
Adjective to adverb	apparent	apparently	rapid	rapidly
Opposite	possibly	impossibly	ethical	unethical
Comparative	great	greater	tough	tougher
Superlative	easy	easiest	lucky	luckiest
Present Participle	think	thinking	read	reading
Nationality adjective	Switzerland	Swiss	Cambodia	Cambodian
Past tense	walking	walked	swimming	swam
Plural nouns	mouse	mice	dollar	dollars
Plural verbs	work	works	speak	speaks

### Count-based vs. Direct Prediction

- Fast training
- Efficient usage of statistics
- Long & Sparse!
  - Length = |V|
  - most elements are zero
- Primarily used to capture word similarity
- Disproportionate importance given to small counts

- Scales with corpus size
- Inefficient usage of statistics
- Short and Dense
  - Length = any hidden size (50-10000)
  - Nearly nothing is zero
- Generate improved performance on other tasks
- Can capture complex patterns beyond word similarity

# Encoding meaning in vector differences

- Key idea:
  - Ratios of co-occurrence probabilities can encode meaning components

Probability and Ratio	k = solid	k = gas	k = water	k = fashion
P(k ice)	$1.9 \times 10^{-4}$	$6.6 \times 10^{-5}$	$3.0 \times 10^{-3}$	$1.7\times10^{-5}$
P(k steam)	$2.2  imes 10^{-5}$	$7.8\times10^{-4}$	$2.2\times10^{-3}$	$1.8\times10^{-5}$
P(k ice)/P(k steam)	8.9	$8.5\times10^{-2}$	1.36	0.96

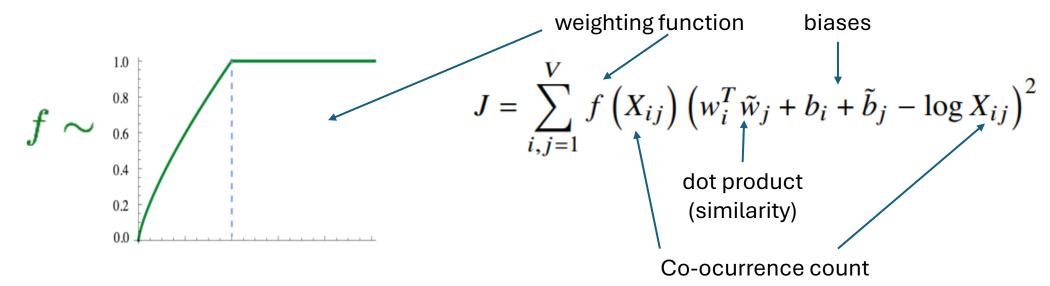
# Encoding meaning in vector differences

- How can we capture ratios of co-occurrence probabilities as meaning components in a word vector space?
- Solution:
  - Log-bilinear model:  $w_i \cdot w_j = \log P(i|j)$
  - with vector differences:  $w_x \cdot (w_a w_b) = \log \frac{P(x|a)}{P(x|b)}$

### GloVe: A new model for learning word representations

$$w_i \cdot w_j = \log P(i|j)$$

$$w_x \cdot (w_a - w_b) = \log \frac{P(x|a)}{P(x|b)}$$



```
# https://github.com/noaRricky/pytorch-glove
class GloVeModel(nn.Module):
    def init (self, embedding size, context size, vocab siz)
        self.focal embeddings = nn.Embedding(
            vocab size, embedding size)
        self.context embeddings = nn.Embedding(
            vocab size, embedding size)
        self.focal biases = nn.Embedding(vocab size, 1)
        self.context biases = nn.Embedding(vocab size, 1)
    def loss(self, focal input, context input, coocurrence count):
                                                                           J = \sum_{i} f(X_{ij}) \left( w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij} \right)^2
        focal embed = self.focal embeddings(focal input)
        context embed = self.context embeddings(context input)
        focal bias = self.focal biases(focal input)
        context bias = self.context biases(context input)
        # count weight factor
        weight factor = torch.pow(coocurrence count / x max, alpha)
        weight factor[weight factor > 1] = 1
        embedding products = torch.sum(focal embed * context embed, dim=1)
        log cooccurrences = torch.log(coocurrence count)
        distance expr = (embedding products + focal bias +
                         context bias + log cooccurrences) ** 2
        single_losses = weight_factor * distance_expr
        mean loss = torch.mean(single losses)
        return mean loss
```

### **Word Similarities**

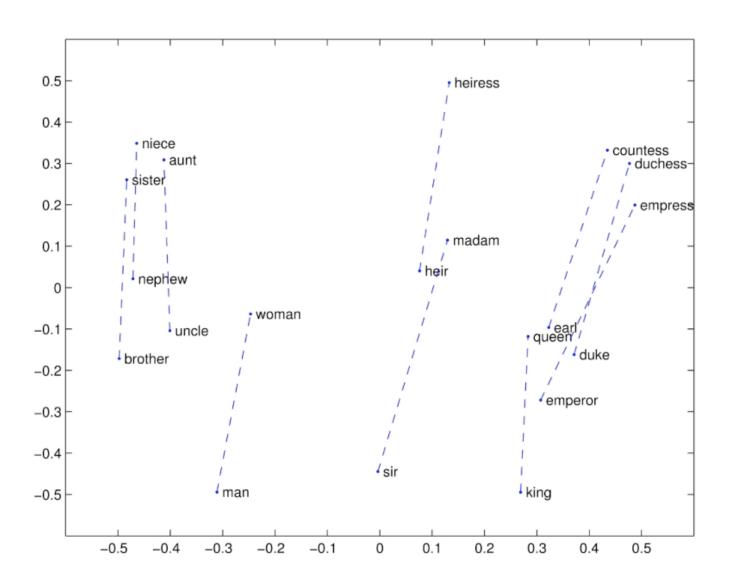
### Nearest words to frog:

- 1. frogs
- 2. toad
- 3. litoria
- 4. leptodactylidae
- 5. rana
- 6. lizard
- 7. eleutherodactylus

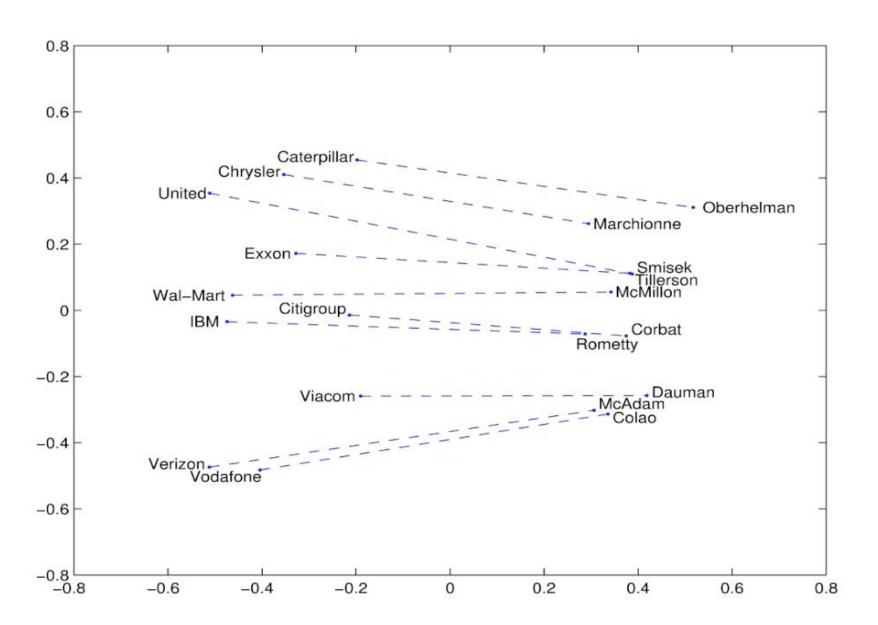




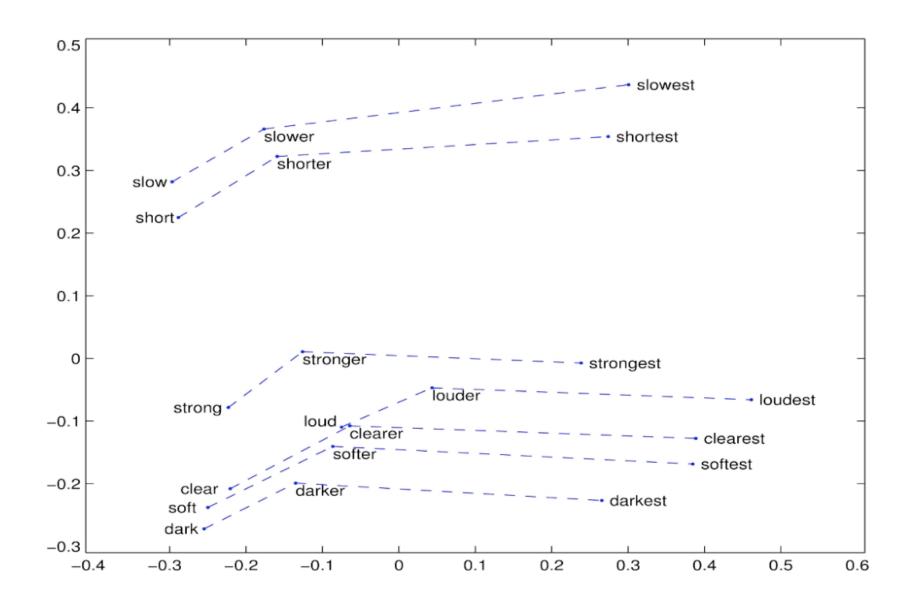
### Linear Structures: Visualizations



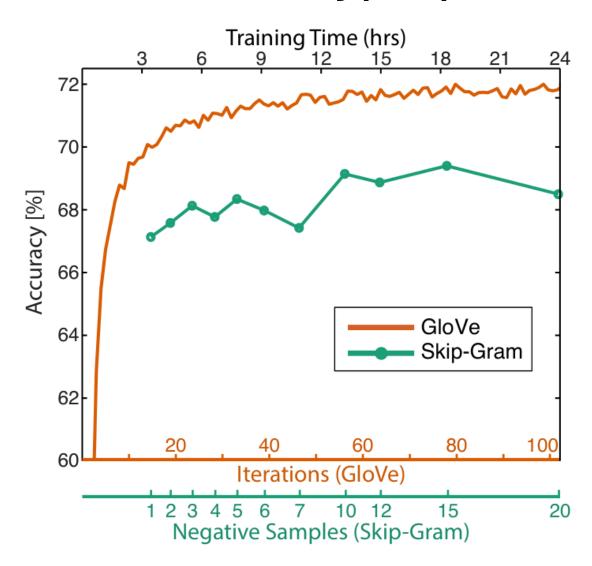
### Linear Structures: Visualizations



### Linear Structures: Visualizations



# Analogy evaluation and hyperparameters



# Word Embedding Conclusion

- Developed a model that can translate meaningful relationships between word-word co-occurrence probabilities into linear relations in the word vector space.
- GloVe shows the connection between Count! work and Predict! work – appropriate scaling of counts gives the properties and performance of Predict! models

# Terminology Hell

- "Embedding"
  - Embedding layer: torch.nn.Embedding
    - Linear layer: one hot index -> vectorized representation
    - Basically, a big look up table
  - Vector(ized) Representation
    - Using an n-dim vector to represent a word. The vector.
  - Hidden Representation; Hidden State
    - The intermediate output of a neural network
    - Neural LM: use this as the vectorized representation
  - Word Embedding:
    - The model/system/algorithm that generate a vectorized representation given a word.
  - Word Embedding:
    - The generated vectorized representation.

adverbs	verbs	adjectives	nouns
appropriately	actualize	24/7	action items
assertively	administrate	24/365	adoption
authoritatively	aggregate	accurate	alignments
collaboratively	architect	adaptive	applications
compellingly	benchmark	agile	architectures
competently	brand	alternative	bandwidth
completely	build	an expanded array	benefits
continually	cloudify	of	best practices
conveniently	communicate	B2B	catalysts for change
credibly	conceptualize	B2C	channels
distinctively	coordinate	backend	clouds
dramatically	create	backward-	collaboration and idea
dynamically	cultivate	compatible	sharing
efficiently	customize	best-of-breed	communities
energistically	deliver	bleeding-edge	content
enthusiastically	deploy	bricks-and-clicks	convergence
fungibly	develop	business	core competencies
globally	dinintermediate	clicks-and-mortar	customer service
holisticly	disseminate	client-based	data

The Corporate B.S. Generator



# Contextual vs. Global Word Embedding

- Global Word Embedding
  - One vector representation word-type
  - word2vec, GloVe
- Contextual Word Embedding
  - One vector representation word-token
  - RNN, LSTM, BERT, GPT...

Recall: Primitives: lexical categories or parts of speech.

- Each word-type is a member of one or more.
- Each word-token is an instance of exactly one.

# Context is important

- WSD:
  - The lawyer approached the bar.
  - Frank approached the bar.
- Anything beyond lexical level:
  - Anaphora:
    - Mary is a doctor. She works at the \_\_\_\_\_
    - Anne is a farmer. She works at the \_\_\_\_\_
  - Long distance agreement:
    - The books that every student have read are ...
- Tasks:
  - What is the capital of Germany. It is \_\_\_\_



Bro conferencing at ACL 2019.

### Also, we need more! What of larger semantic units?

- How can we know when larger units are similar in meaning?
  - *CTV News*: Poilievre-led attempt to bring down Trudeau minority over carbon tax fails.
  - CBC News: Liberals survive non-confidence vote on carbon tax with Bloc, NDP backing.
  - The Beaverton: Co-worker that everyone hates surprised he can't get colleagues to do what he wants.



NATIONAL - 2 WEEKS AGO

Co-worker that everyone hates surprised he can't get colleagues to do what he wants

OTTAWA – Local man Pierre Poilievre, an employee at an Ottawa small business named the House of Commons, was surprised that none of the colleagues who despise him were willing to support hi...

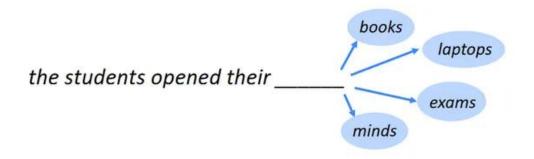


# Language Modelling Task

• Final goal: predict/estimate the probability of a sequence

### Probability (Some sentence over here.)

- Language Models:
  - Estimate the likelihood of sequence of texts.
  - Model that assigns probabilities to sequences of words.
  - (All good definitions)
- Actual task:
  - Predict the next word
  - MLM



# Likelihood, Probability of What?

- A really vague, abstract notion of how likely this sentence can appear in real-world, natural language discourse.
- Ultimate Goal: create a statistical model to describe some large quantity of corpora data, that ultimately capture the statistical structure of "Language."
- Again, the practical definition of Language Model is:
  - Estimate the likelihood of sequence of texts.
  - Model that assigns probabilities to sequences of words.

(both are fine)

- Likelihood is not: (related terms)
  - Grammaticality, acceptability, syntactic well-formedness, "make sense," well written ...
  - Colorless green ideas sleep furiously. -> Chomsky was terribly wrong.

#### The Language Modelling Pipeline

- Collect large quantity of unstructured data
  - Wikipedia articles, social media post, news articles...
  - Famous open-source: WikiText-2/103 (100M Tokens), Dolma (3T tokens)

#### Tokenization

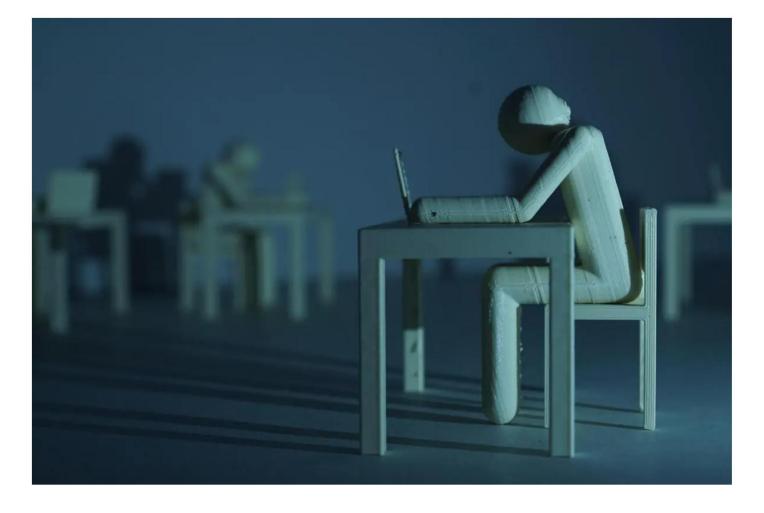
- The Technische Universität Darmstadt, commonly known as TU Darmstadt, is a public research university in the city of Darmstadt, Germany.
- ['The', 'ĠTechn', 'ische', 'ĠUnivers', 'it', 'ä', 't', 'ĠD', 'arm', 'stadt', ',', 'Ġcommonly', 'Ġknown', 'Ġas', 'ĠT', 'U', 'ĠD', 'arm', 'stadt', ',', 'Ġis', 'Ġa', 'Ġpublic', 'Ġresearch', 'Ġuniversity', 'Ġin', 'Ġthe', 'Ġcity', 'Ġof', 'ĠD', 'arm', 'stadt', ',', 'ĠGermany', '.']
- Train the actual model by performing the language modelling task:
  - Next Word Prediction, Masked Language Modelling, ...

#### Language Modelling is **NOT Unsupervised!**

- Supervised vs. unsupervised learning.
  - Supervised learning: labelled data.
  - Unsupervised learning: unlabelled data, find structure in data itself.
    - E.g., Clustering (e.g., K-Means) and Dimensionality Reduction (e.g., PCA, t-SNE).
- Language modelling tasks require direct supervision.
  - Label: next token.
- Why many people call Language Modelling unsupervised?
  - Data collection: unprocessed, unstructured raw text from the internet.
  - Basically free!
  - Indeed, we may not need to hire people to do any labelling or annotation.

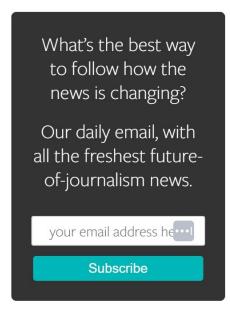
#### Language Modelling is **NOT Unsupervised!**

- Why many people call Language Modelling unsupervised?
  - Data collection: unprocessed, unstructured raw text from the internet.
  - Basically free!
  - Indeed, we may not need to hire people to do any labelling or annotation.
- No longer the case now (2023, 2024ish)!
  - The major LLM research labs (Qwen, Llama, OpenAI...) have used nearly all the data human ever created.
  - Most commercial models need large amount of curated data.



# Meet the journalists training AI models for Meta and OpenAI

The gig work platform Outlier is one of several companies courting journalists to train large language models (LLMs).



#### THE LATEST FROM NIEMAN LAB



BBC resignations are a symbol of mounting pressure on public broadcasters in the Trump era

**DENIS MULLER** 

#### **Tokenization and Tokenizers**

- Character-level language modeling:
  - Classifying Names with a Character-Level RNN
  - Good with Chinese
  - Other languages: inefficient use of data
- Tokenization: breaks down text into smaller units, often called tokens.
  - text.split()
- The only difficulty: unknown token.
  - Special <unk > token
     #longexposurephotogram

#longexposurephotography
Long exposure photography

Rechtsschutzversicherungsgesellschaft Rechts Schutz Versicherung s Gesellschaft legal protection insurance company

# Agglutinative language





Eiskaffee



Çekoslovakya - Czechia(in ussr)

-(l)ı : from

-(l)aştır : to (let it) become

-(a)ma : not

-dı: past

-k: 1st pronoun(us)

-lar: plural

-(ı)mız: (from) us

-dan: out of /from

-mış: past (heard)

-sı(n)ız: you



Çekoslovakyalılaştıramadıklarımızdanmışsınız

"You are reportedly one of those that we could not make Czechoslovakian."

#### **Tokenization and Tokenizers**

Solution: Break a word down into subwords, or word pieces!

- Slightly different encoding styles colorless green ideas sleep furiously
  - BERT: 'color', '##less', 'green', 'ideas', 'sleep', 'furiously'
  - GPT/LLaMA: 'color', 'less', 'Ġgreen', 'Ġideas', 'Ġsleep', 'Ġfuriously'
  - XLM: 'color', 'less</w>', 'green</w>', 'ideas</w>', 'sleep</w>', 'furiously</w>'

- Subword tokenization technique.
- Used for data compression and dealing with unknown words.
- Initialization:
- Vocabulary = set of all individual characters.
- $V = \{A, B, C, ..., a, b, c, ..., 1, 2, 3, ..., 1, \$, \%, ...\}$
- Repeat:
  - Choose two symbols that appear as a pair most frequently (say "a" and "t").
  - Add new merged symbol ("at").
  - Replace each occurrence with the new symbol ("t","h","a","t" -> "t","h","at").
- Until k merges have been done. Usually, we select a pre-defined vocabulary size (k).

```
Segments:
                               Vocabulary:
      1 o w </w>
                               </w>>, d, e, I, l, n, o, r, s, t, w
  lowest</w>
  newer</w>
  wider</w>
   n e w </w>
Most frequent symbol pair: er (9 times)
Segments:
                               Vocabulary:
      1 o w </w>
                               </w>, d, e, I, l, n, o, r, s, t, w, er
      lowest </w>
   n e w er </w>
    wider</w>
    n e w </w>
```

```
Segments:
                                 Vocabulary:
      1 o w </w>
                                 </w>>, d, e, I, l, n, o, r, s, t, w, er
  1 o w e s t </w>
  n e w er </w>
  wider</w>
    n e w </w>
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Segments:
                                 Vocabulary:
      1 o w </w>
                                 </w>, d, e, I, l, n, o, r, s, t, w, er,
      lowest </w>
                                 er</w>
   n e w er</w>
    w i d er</w>
     n e w </w>
```

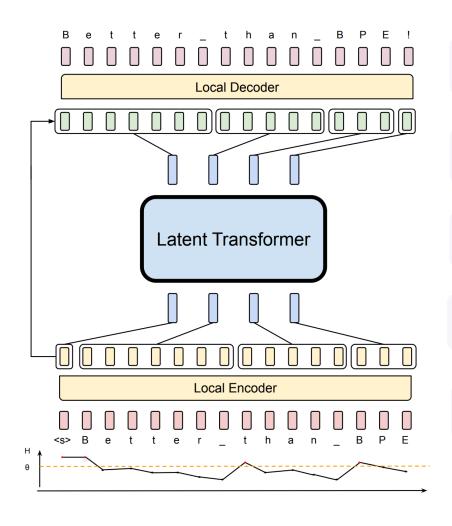
```
Segments:
                                  Vocabulary:
      1 \circ w < /w >
                                  </w>, d, e, I, l, n, o, r, s, t, w, er,
  lowest</w>
                                  er</w>
  n e w er</w>
  w i d er</w>
    n e w </w>
Most frequent symbol pair: ne (8 times)
Segments:
                                  Vocabulary:
      1 \circ w < /w >
                                  </w>, d, e, I, l, n, o, r, s, t, w, er,
      lowest </w>
                                  er</w>, ne
   ne w er</w>
    w i d er</w>
     ne w </w>
```

#### Limitations

- Doesn't work well for other writing systems:
  - Especially Arabic.
- Typos:
  - participating -> 'Ġparticipating'
  - partcipating -> 'Ġpart', 'c', 'ip', 'ating'
- "How many r's does the word strawberry has?"

#### Possible Future Directions

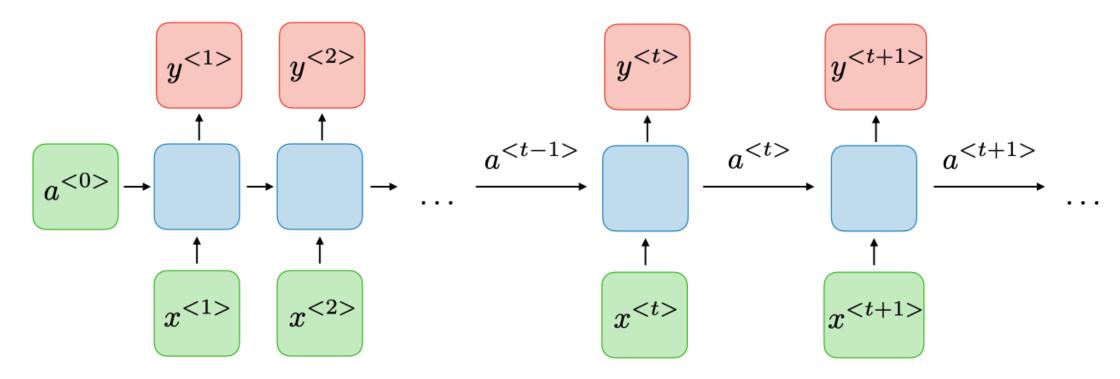
- Byte-based tokenization
  - Byte Latent Transformer
  - Pagnoni et al. (2025).
- Language (writing system) specific tokenization.
  - E.g., Arabic: morphological methods.
- Modularity & model merging: combine models together. (More on this later)



- Small Byte-Level Transformer Makes Next-Byte Prediction
- 4. **Unpatching** to Byte Sequence via Cross-Attn
- 3. Large Latent Transformer **Predicts Next Patch**
- 2. Entropy-Based Grouping of Bytes Into **Patches** via Cross-Attn
- Byte-Level Small Transformer Encodes Byte Stream

#### Training a Language Model

• Let's take recurrent neural network (RNN) as an example.

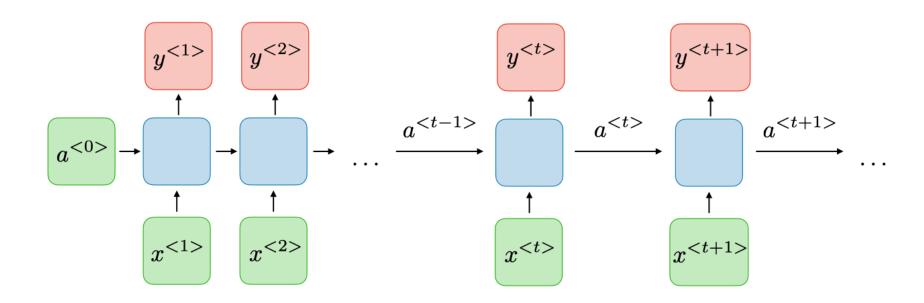


$$\mathbf{x} \leftarrow f(\mathbf{x}\mathbf{W} + \mathbf{b})$$

$$\mathbf{a}_i \leftarrow f(\mathbf{x}_i \mathbf{W}_x + \mathbf{a}_{i-1} \mathbf{W}_a + \mathbf{b})$$

Input token representation

RNN hidden state



$$y_i \leftarrow \mathbf{a}_i \mathbf{W}_{out}$$

```
class RNN(nn.Module):
   def init (self, input size, hidden size, output size):
       # i: input token, h: hidden state, o: output
       self.i2h = nn.Embedding(input_size, hidden_size)
       self.h2h = nn.Linear(hidden size, hidden size)
       self.h2o = nn.Linear(hidden_size, output_size) # output_size: number of labels
   def forward(self, x, hidden state):
       x = self.i2h(x)
       hidden state = self.h2h(hidden state)
       hidden_state = torch.tanh(x + hidden_state)
       out = self.h2o(hidden_state)
```

return out, hidden state

- Model:
  - Let's take this RNN model as an example.
- Data:
  - A corpus with sentences like: a tablespoon of apricot jam
- Input:
  - a tablespoon of apricot
- Output label:
  - jam

- Model:
  - Let's take this RNN model as an example.
- Data:
  - A corpus with sentences like: a tablespoon of apricot jam
- Input:
  - a tablespoon of
- Output label:
  - apricot

- Model:
  - Let's take this RNN model as an example.
- Data:
  - A corpus with sentences like: a tablespoon of apricot jam
- Input:
  - a tablespoon
- Output label:
  - of

- Model:
  - Let's take this RNN model as an example.
- Data:
  - A corpus with sentences like: a tablespoon of apricot jam
- Input:
  - a
- Output label:
  - tablespoon

#### Summary: How to train a neural LM

- Collect training data.
- Train the model:
  - Take a tokenized sequence
  - Input:  $t_1, t_2, t_3, ..., t_{n-1}$
  - Output: logits (or probability) of t<sub>n</sub>.
- The LM is typically a neural model,
  - trained as a typical neural model.